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ABSTRACT

On the basis of studies reported elsewhere (SE 019 717 and 719) the author discusses the implications of research on cognitive development for teachers whose subjects involve an understanding of proportionality. Several common methods of teaching chemistry students to use proportionality in chemical problems are described. Research on the development of proportionality concepts is discussed, and basic results of Inhelder and Piaget, Karplus, the author, and others are briefly outlined. Laboratory tasks which can help students understand proportionality are described, and cooperation between teachers and psychologists is advocated. (SD)

Proportionality: As Seen by Psychologists and Teachers

by

Susan Abramowitz

ABSTRACT

Chemistry teachers often complain of the difficulty their students have handling scientific concepts that are expressed mathematically. The methods that teachers use to help their students understand problems involving stoichiometric and gas law relations are described. The assumption is made that these problems are difficult because they demand an understanding of proportionality. Both developmental and educational psychologists have investigated how children come to understand proportionality. A brief discussion of their work and the relevance it has for classroom teachers is followed by suggestions that classroom teachers, themselves, might use to help students understand proportionality.

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A long-standing complaint among chemistry teachers is that their students are ill-prepared mathematically to handle problems involving stoichiometric and gas law relations. Stoichiometric relations are chemical recipes which help the chemist relate theoretical quantities to quantities in the microscopic world. The coefficients in the following equation specify the number of "moles" of ingrediants necessary to produce the number of "moles" of water.

$$2H_2 + O_2 = 2H_2O$$

Each mole of hydrogen and oxygen has a characteristic mass and volume. An example of a stoichiometric problem would be, "Determine the amount of water that ten grams of hydrogen would produce." Using the characteristic masses of 2 grams of hydrogen produce 18 grams of water, we arrive at an answer that 10 grams of hydrogen produce 90 grams of water.

Gases behave ideally in such a way that the state of any gas sample may be characterized by any three of the following four variables: volume (V), pressure (P), temperature (T), and quantity of moles (n). This characterization can be expressed mathematically as $PV = nRT$, where the value of "R" is constant. Questions about gas laws involve asking the student to determine the value of any one of the parameters, given the values of all the others.

In order to teach these topics, teachers have devised countless methods to facilitate the student's understanding of the scientific concepts and their mathematical formulation. One method used is called the "factor-label" method of dimensional analysis. When asked to determine how much 10 moles of hydrogen gas weigh, the student might set up the following relationship so that all labels cancel out except the one that specifies the quantity

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asked for.

$$(\text{quantity of } H_2) \text{ moles} \times (H_2 \text{ mass per unit quantity}) \text{ grams/moles} = (H_2 \text{ mass}) \text{ grams}$$

Another method teachers use is to ask the student to reason from knowledge of physical relations. For example, given a finite amount of oxygen gas the temperature of which is raised from 273 degrees Kelvin to 546 degrees Kelvin, what will the pressure be at the higher temperature, if the initial pressure was 1 atmosphere? In this problem, an increase in the temperature of the gas will cause its pressure to increase. Therefore, the resultant pressure will be greater than the initial pressure. Thus one must multiply the initial pressure by a ratio of temperatures that is larger than one.

$$(546^\circ \text{ K}/ 273^\circ \text{ K}) \times 1 \text{ atmosphere} = 2 \text{ atmospheres}$$

The final method to be illustrated is one that is often used as a last resort. Teachers merely provide the student with a battery of formulae, such as

$$P_1V_1/ T_1 = P_2V_2/ T_2$$

where the "1" and "2" signal the initial and final state respectively of the gas.

Regardless of the methods teachers devise, they are ultimately frustrated by the inability of many of their students to solve these problems successfully. One way of improving the situation is to have students take the proper mathematics courses before they take their science course. Another is to integrate the teaching of mathematics with chemistry. The former depends on the vicissitudes of school programming and the latter on the creation of new curricula. But even if programing were perfect and integrated courses were available, student's difficulty with these problems is likely to persist until the psychological determinants of the difficulty are discovered and effective remediation developed.

It is the author's belief that students have difficulty with these problems because they demand an understanding of proportionality, a concept that not only is difficult to comprehend and operationalize, but also is basic to all the above methods of problem solving. The remainder of the paper will be devoted to a discussion of the potential psychological research has for helping us understand why children have problems with the concept of proportionality as well as the implications this research has for us as teachers.

PROPORTIONALITY: A DEVELOPMENTAL CONCEPT?

Research into this problem has been done mostly within a developmental framework. Developmental psychologists assume that children progress through discrete and characteristic stages as they come to understand a concept. Children of different ages are given a set of tasks to perform. The performance of children in each age group is compared and inferences are made as to how an understanding of the concept develops from one age to the next.

One of the most prominant psychologists in this field is Piaget, who has postulated that it isn't until one is between the ages of 12 and 13 that one has the cognitive skills which are necessary for an understanding of proportionality (1958)[1]. He believes that the necessary skills depend on the ability not only to compensate for changes in proportional relationships, but also to do so metrically. For example, not only would an adolescent have to know that the larger weight on a balance at a shorter distance from the fulcrum would be offset by a smaller weight at a larger distance, but also how to determine the quantitative value of this compensation. As suggestive as Piaget's formulation is, it has been exceptionally difficult to give empirical content to the logico-mathematical elements of his theory.

Another researcher in this area is Robert Karplus, who has isolated various strategies that children use to explain their answers to a proportion problem (1970[2]; 1972[3]). The problem he gave his subjects involved two stick figures of different heights. The stick figures were first measured with large paper clips. Then only one of the figures was measured with small paper clips. The children then had to predict the height of the other stick figure as measured in small paper clips.

In analyzing the responses of 4th to 8th graders, Karplus isolated a developmental sequence. He found that the younger children tended to make predictions by guessing on the basis of appearances and used the numbers they were given haphazardly. The children in the middle age range used two different kinds of strategies. They either used all the data by computing the difference between two of the measurements and adding the difference to the third measurement (a method in which the ratios do not figure) or they used a scaling factor (such as multiplying by 2) which was not related to the ratio inherent in the measurements. The older children tended to use either a combination of the latter two strategies or were able to solve the problem using a proportion. Thus Karplus's work indicates that children of different ages approach this problem in different ways.

Other researchers have looked at the problem of proportionality from a practical rather than a developmental perspective. Steffe and Parr (1968)[4] tried to partition out the potential cause of difficulty due to the mode of presentation of a proportion problem. They investigated the differential success of 6th graders with proportions expressed as "ratios" (comparison of two sets of different objects by division) and "fractions" (comparison of different amounts of similar objects by division). They also varied the position of the unknown in these problems.

Other studies have isolated factors besides age which affect the attainment of an understanding of proportionality. The kinds of problems children are given[5], the apparatus that is used[6], the learning environment to which children have been exposed[7], as well as ability as measured by an intelligence test are all factors which influence how well children perform on tests involving proportionality.

But there is still much work that must be done before we really understand why children have difficulty with the concept of proportionality. Studies done exclusively within a cross-sectional developmental framework are limited. Although one may establish the presence of certain solution forms in various age groups, generalization to individual development is suspect. Even if a developmental sequence were known, it would only specify stages through which a child may proceed without giving any indication as to what someone early in the sequence is lacking that prevents the appearance of a mature solution.

CAN STUDENTS BE TRAINED TO UNDERSTAND PROPORTIONALITY?

A way of getting around the limitations of developmental studies is to run experiments in which subjects are trained to solve proportion problems. An example of a training task is one in which subjects are asked to predict the answer to a proportion problem involving measurement, and then check out their prediction by actually measuring the object. If their prediction and measurement values did not coincide, they could be asked to think of reasons for the discrepancy and ways to avoid such discrepancies in the future. Thus a training task would be set up in which subjects would devise strategies for predicting the correct solution. Finally, how well subjects can use their newly devised strategies in tasks different than the one on which they were trained would be tested. If training in the lab were successful, it would offer suggestions as how one might proceed in the real world of the classroom.

Teachers have ample opportunity to conduct experiments of this sort even though they may lack controls that an experimental psychologist may employ. When exisitng teaching techniques are not successful, teachers are expected to try out new ones, and "subjects" for these trials are readily available. The possibility exists also that the methods teachers devise may shed some light on how students develop an understanding of proportionality.

Most chemistry teachers "cover" the metric system. This topic gives them an opportunity to provide their students with experiences which build up their ability to handle proportion problems. For example, they can ask students to measure and compare the lengths of two different line segments with a lined index card. Then the students could repeat their measurements and comparisons only this time with a strip of ruled paper. The aim of such a task is to give children the chance to observe the constancy of relations regardless of the measuring device used. Also, it enables students to practice with and develop from concrete experiences a working vocabulary with which to express relationships.

Another task is to give students an assignment to measure various objects in the classroom, such as a table top, door, and window, with a lined index card. Students may find suitable replacements, such as their feet or a long rod, with which to more easily measure these objects. From a task like this, they may see the need of a measuring device that is made up of smaller units. They are also able to develop the idea of converting from larger to smaller units and vice versa.

A third task of this sort is to measure a pair of line segments, and then a third segment, with a lined index card. The students' assignment is to determine the length of a fourth line segment so that a second pair is constructed with the same relationship between its members as was seen in the first pair. In order to solve these problems, students have to recognize the relationship between any two of the three given line segments. Using this relation, they can then figure out what the length of the fourth line segment must be. Such a task provides a concrete experience with relationships involving proportionality.

As a chemistry teacher, I made use of the many opportunities I had to help my students broaden their understanding of proportional relationships once the initial foundation had been laid with the unit on measuring. Throughout the year, every effort was made to increase their facility with mathematical relations which were directly applicable to chemical concepts. To help my students with conversion problems involving the metric system, the concept that equality is maintained by multiplying any number by the quantity "1" was presented.

To facilitate the understanding of stoichiometric relations, I constructured behavioral objectives, which broke down the learning process into more manageable sequential steps. Thus 'To calculate weight-weight problems' was a task near the end of a sequence which began with 'To calculate gram molecular weights', followed by 'To calculate the number of moles in 10 grams of the following', etc. For the first time in my years of teaching, students were able to manipulate data they gathered from labs involving these relationships with a minimum of difficulty. This is because the objective 'To calculate weight-weight relations from lab data' was the final objective in the series of behavioral objectives which helped the students to master the topic in gradual incremental steps.

To determine whether my students had improved in their understanding of proportionality, I administered the Karplus task of proportionality at the beginning and end of the school year. There was a marked shift in their performance. Whereas the students at the beginning of the year had overwhelmingly solved the task using strategies unrelated to proportionality, by the end of the year my students were using more sophisticated strategies which involved the ratios of numbers.

TEACHERS AND PSYCHOLOGISTS CAN HELP EACH OTHER

We have now come full circle from the complaints chemistry teachers have made about their students' inability to solve stoichiometric and gas law problems, both of which demand an understanding of proportionality. From developmental psychologists, we have learned that adolescence marks the beginning of a new stage of cognitive growth. During this period adolescents are beginning to develop the skill of abstract reasoning. They are learning how to look at the world from many points of view. Because these new capabilities are developing, they may have difficulty integrating mathematical techniques they had once learned with their application to chemistry problems.

Moreover, techniques used to determine how a child may be trained to learn proportionality were seen to have direct relevance for the classroom teacher. For it is in the classroom that teachers with imagination and flexibility can create tasks which allow students to build up an understanding of proportionality throughout the school year. Once such a foundation is laid, students may have less difficulty solving chemistry problems that must be translated into the language of mathematics. Lastly, it is possible that the techniques teachers devise may contribute to our understanding of how children develop the concept of proportionality.

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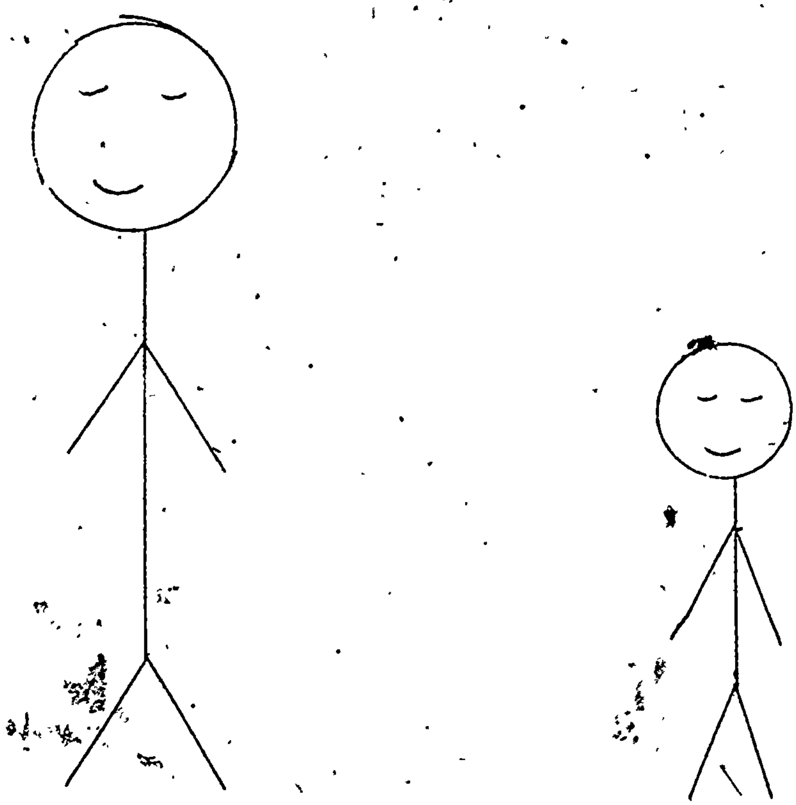
Appendix A

The Karplus Task of Proportionality



Mr. Tall

6 Biggies

Mr. Short

4 Biggies, 6 Smallies

Question: Predict the height of Mr. Tall as measured by "Smallies"

Explain how you arrived at your prediction in the space below.

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